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# Analytical calculation based on probability model for the pattern effect of SOA as in-line amplifier for WDM system

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## Abstract

A probability model is developed to calculate the intensity and phase noise induced by a semiconductor optical amplifier used as an in-line amplifier in a WDM system. An analytical result is obtained which shows the dependence of noise on number of channels, input power, bit rate, and carrier lifetime. The result shows that adding a CW reservoir channel is an efficient way to suppress the intensity noise. The theoretical results are in agreement with numeric simulations and reported experiments. © 2001 Published by Elsevier Science B.V.

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## 1. Introduction

Semiconductor optical amplifiers (SOAs) are very attractive for their wide gain spectrum, capability of integration with other devices, and potential low cost. Also, much research activities has been done on all-optical signal processing with SOAs [1,2]. Though the SOA was formerly considered to be unsuitable as an in-line amplifier, due to its pattern effect induced by the carrier recovery, it is possible to use it in a multi-channel WDM transmission system. This is because the relative variation of the total power, which determines the severity of the pattern effect, decreases in WDM systems comparing to single channel systems. In recent years, there were a number of DWDM

transmission experiments successfully performed with SOAs as in-line amplifiers. Among these experiments, there are the transmission of  $32 \times 2.5$  Gbit/s signal over 315 km of AllWave™ fiber with three SOAs [3], the transmission of  $8 \times 40$  and  $32 \times 10$  Gbit/s signals over 160 km of standard fiber with four SOAs [4], and the transmission of  $8 \times 20$  Gbit/s signal over 160 km of standard fiber with seven SOAs [5]. The experiments show that the power penalty induced by the pattern effect of the SOAs increases exponentially with the increase of input or output power of the SOAs. Ref. [3] also shows that the use of reservoir channel in optical networks with SOAs is effective for achieving high transmission quality. However, to our knowledge, the detailed theoretical analysis of these phenomena have not been put forward.

In this paper, we will theoretically analyze one stage of SOA in-line amplification in a DWDM transmission system. The gain dynamics of a SOA responding to a single pulse has been well studied

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[1,6,7], and signal statistics under the static situation have been put forward [8]. However, to our knowledge, this is the first time the method to analyze the gain dynamics under random bit-stream condition and obtaining analytic results has been presented.

For a random signal stream in WDM systems, the dynamic and probability models are first developed in Section 2 to deal with the pattern effect. An analytic result on the power penalty and chirp induced by the SOA is then obtained in Section 3 to show their dependence on different bit rates, number of channels and input powers. These results are applied to get useful curves for typical systems in Section 4, and the results of numeric simulations are also shown on these curves. Section 5 contains further discussion on the veracity of the analytical results. Finally, the effect of the reservoir channel is examined theoretically in Section 6.

## 2. Model explanation

### 2.1. Dynamic equation

The calculation in this paper is based on the dynamic model developed in Ref. [6] with the differential equation

$$\frac{dh}{d\tau} = \frac{g_0 L - h}{\tau_c} - \frac{P_{in}(\tau)}{E_{sat}} [\exp(h) - 1] \quad (1)$$

where  $\tau_c$  is the carrier lifetime,  $E_{sat}$  is the saturation energy,  $P_{in}$  is the total input light power,  $g_0$  is the small-signal gain,  $L$  is the length of the SOA, and  $h(\tau)$  is the integrated gain defined as

$$h(\tau) = \int_0^L g(z, \tau) dz \quad (2)$$

and  $\tau = t - z/v_g$  is a coordinate in a reference frame moving with the signal.

When the number of channels  $N$  is large enough, the total input light power  $P_{in}$  lies around the average value mostly and so does the integrated gain  $h(\tau)$ . So, they can be written as

$$P_{in}(\tau) = \bar{P}_{in} + \Delta P_{in}(\tau) \quad (3)$$

$$h(\tau) = \bar{h} + \Delta h(\tau) \quad (4)$$

where,  $\Delta P_{in}(\tau)$  and  $\Delta h(\tau)$  are considered to be small disturbance of the system.

Then, we can get from Eq. (1), by neglecting the second and higher order disturbance, that

$$\frac{d\Delta h}{d\tau} + \frac{\Delta h}{\tau'_c} = -\frac{\Delta P_{in}}{E_{sat}} [\exp(\bar{h}) - 1] \quad (5)$$

$$\text{where } \frac{1}{\tau'_c} = \frac{1}{\tau_c} + \frac{P_{in} \exp(\bar{h})}{E_{sat}}$$

and  $\tau'_c$  is the effective carrier lifetime.

Multiplying both sides of Eq. (6) with  $\exp(\tau'/\tau'_c)$  and integrating from  $-\infty$  to  $\tau$  gives

$$\Delta h(\tau) = K \int_{-\infty}^{\tau} \Delta P_{in}(\tau') \exp\left(\frac{\tau' - \tau}{\tau'_c}\right) d\tau' \quad (6)$$

where

$$K = \frac{1 - \exp(\bar{h})}{E_{sat}} < 0 \quad (7)$$

Eq. (6) is the basic dynamic equation used in this paper.

### 2.2. Probability representation

When a multi-channel signal is concerned, assuming that each channel is coded with an ideal rectangular NRZ pulse, the total input power of  $N$  channels has a binomial probability distribution. If the average input power per channel is  $p$ , the probability that the total power equals  $mp$  is

$$\Pr\{P_{in} = mp\} = \binom{N}{m} \times \left(\frac{1}{2}\right)^N \quad (8)$$

where  $m = 1, 2, \dots, N$ , and

$$E[P_{in}] = Np \quad (9)$$

When the channels are completely asynchronous, the total power of input signal can be regarded as a steady-state ergodic process. By further assuming that any two bits are independent, the autocorrelation function of  $\Delta P_{in}(\tau)$  can be found to be

$$R_{\Delta p}(t-s) = E[\Delta P_{\text{in}}(t)\Delta P_{\text{in}}(s)] \\ = \begin{cases} Np^2 \left(1 - \frac{|t-s|}{T}\right), & |t-s| < T \\ 0, & |t-s| > T \end{cases} \quad (10)$$

Given the probability representation of the input signal and the dynamic equation (6), the statistical properties of the gain and the output signal can be obtained.

### 3. Analytical results

#### 3.1. Integrated gain $\Delta h$

From the theory of stochastic process, the autocorrelation function of the disturbance of integrated gain  $\Delta h(\tau)$  can be derived from dynamic equation (6) and disturbance of input signal (10), which is

$$R_{\Delta h}(t-s) \\ = \begin{cases} NK^2 p^2 \frac{\tau_c^3}{2T} \left[ \exp\left(-\frac{T}{\tau_c}\right) + \exp\left(\frac{T}{\tau_c}\right) - 2 \right] \\ \quad \times \exp\left(-\frac{|t-s|}{\tau_c}\right), & |t-s| > T \\ NK^2 p^2 \left\{ \tau_c^2 - \frac{\tau_c^2}{T} |t-s| + \frac{\tau_c^3}{2T} \left[ \exp\left(-\frac{T+|t-s|}{\tau_c}\right) \right. \right. \\ \quad \left. \left. + \exp\left(-\frac{T-|t-s|}{\tau_c}\right) \right. \right. \\ \quad \left. \left. - 2 \exp\left(-\frac{|t-s|}{\tau_c}\right) \right] \right\}, & |t-s| < T \end{cases} \quad (11)$$

In particular, when  $t$  equals  $s$ , Eq. (11) gives the variance of  $\Delta h(\tau)$

$$\delta^2 = V_{\Delta v} \\ = \frac{P_{\text{os}}^2 \tau_c^2}{NP_{\text{sat}}^2 \tau_c^2} \left\{ 1 + \frac{\tau_c}{T} \left[ \exp\left(-\frac{T}{\tau_c}\right) - 1 \right] \right\} \quad (12)$$

where  $P_{\text{os}} = [\exp(\bar{h}) - 1] \cdot Np$  is approximately the average total output signal power and  $P_{\text{sat}} = E_{\text{sat}}/\tau_c$  is the saturated power.

#### 3.2. Intensity noise

In order to derive the statistical properties of gain  $G = \exp(h)$ , we must know the distribution of

$\Delta h(\tau)$ . Since  $\Delta h(\tau)$  comes from the sum of many independent random variables, as shown in dynamic equation (6), and each one of the variables obeys a binomial distribution which approaches the norm distribution when the number of channels  $N$  is large enough, the distribution of  $h(\tau)$  should be close to a Gaussian distribution. This is verified by the numerical simulations, as will be shown in Section 5. So, the expected value and variance of gain  $G$  can be found to be

$$E(G) = \exp\left(\bar{h} + \frac{\delta^2}{2}\right) \quad (13)$$

$$V_G = \exp(2\bar{h} + 2\delta^2)[1 - \exp(-\delta^2)] \quad (14)$$

From the system viewpoint, the fluctuation of gain  $\Delta G$  will introduce intensity noise into the system. The relative amplitude of the intensity noise is

$$r_1 = \frac{\sqrt{E(\Delta G^2)}}{E(G)} = [\exp(\delta^2) - 1]^{1/2} \quad (15)$$

For a receiver with an optimized power decision threshold, the power penalty induced by the intensity noise is [9]

$$\Delta P = -10 \log_{10} \{1 - [\exp(\delta^2) - 1]Q^2\} \quad (16)$$

#### 3.3. Chirp

The instantaneous frequency of the output signal varies with integrated gain  $h(\tau)$ , and the induced chirp  $\Delta v(\tau)$  obeys

$$\Delta v_{\text{out}}(\tau) = \Delta v_{\text{in}}(\tau) + \frac{\alpha}{4\pi} \frac{\partial h}{\partial \tau} \quad (17)$$

where  $\alpha$  is the line-width enhancement factor [6].

So, in the small disturbance condition and under the assumption that input signal has no chirp, we can obtain, from Eqs. (17) and (1), the variation of the temporary frequency as

$$V_{\Delta v} = \frac{\alpha^2 P_{\text{os}}^2 \tau_c^2}{16\pi^2 NP_{\text{sat}}^2 \tau_c^2} \left\{ 4 + 3 \frac{\tau_c}{T} \left[ \exp\left(-\frac{T}{\tau_c}\right) - 1 \right] \right\} \quad (18)$$

4. Calculated curves

4.1. The calculated power penalty

In order to apply the theory into practical systems, we assumed a typical SOA with 30 dB small-signal gain, 300 ps carrier lifetime and 3 pJ saturate energy. The saturate power of such a device is  $P_{\text{sat}} = 10$  dBm. By applying Eq. (15), we can get the amplitude of intensity noise induced by this SOA versus its output optical power in  $8 \times 2.5$ ,  $8 \times 10$ ,  $16 \times 10$  and  $32 \times 10$  Gbit/s DWDM systems, respectively. These curves are shown in Fig. 1. In this figure, the numeric simulation results are marked. We can see that the analytical and simulated results correspond to each other very well, especially in the small intensity noise region (relative intensity noise  $< 0.07$ ). In the large intensity noise region, slight differences exist between the numeric and analytic results. It is because, in this region, the large intensity noise corresponds to wide gain distribution, and the small-variation approximation made in Eq. (6) does not hold tightly.

Through Eq. (16), we can get the power penalties corresponding to the intensity noise in Fig. 1, as shown in Fig. 2.

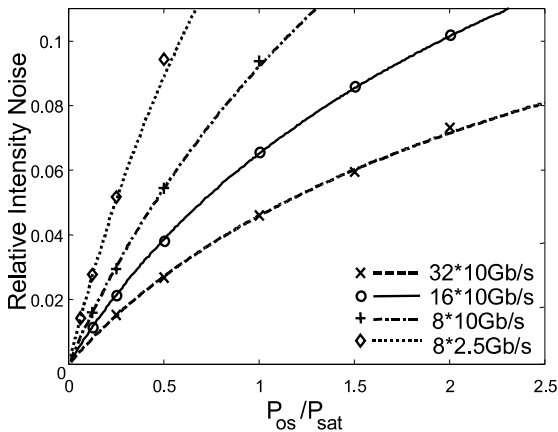


Fig. 1. The relative amplitude of intensity noise  $r_1$  versus total output power  $P_{\text{os}}$  on different conditions. The curves are calculated with Eq. (15) and the marks are results of numeric simulations. The numbers of channels and bit rates corresponding to the curves and marks are shown in the figure.

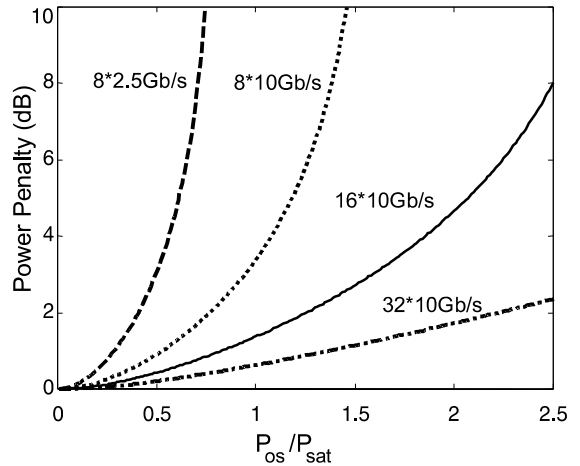


Fig. 2. Power penalty versus total output power  $P_{\text{os}}$  on different conditions. The number of channels and bit rate of every curve are shown beside the curve.

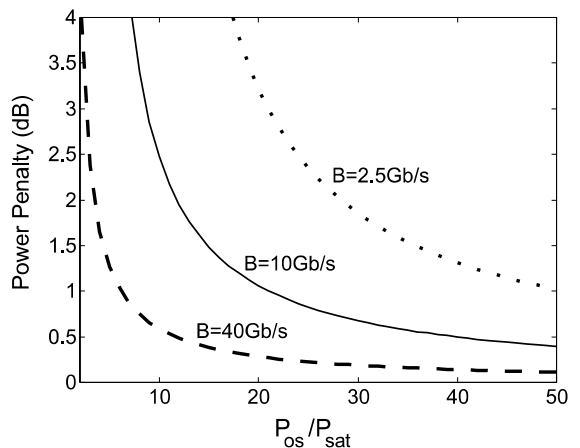


Fig. 3. Power penalty versus number of channels  $N$  in different bit rate  $B$  when the output signal power equals saturated power.

Fig. 3 shows the dependence of power penalty on the number of channels  $N$  and bit rate  $B$ , in the condition that the total output power equals the saturated power of the device. From the figure we can see that, for a system with a given bit rate, low power penalty can be obtained when the channel number exceeds some threshold value. And this threshold value decreases with the increase of bit rate, taking approximate value of 30, 20, and 10 for a 2.5, 10 and 40 Gbit/s system respectively.

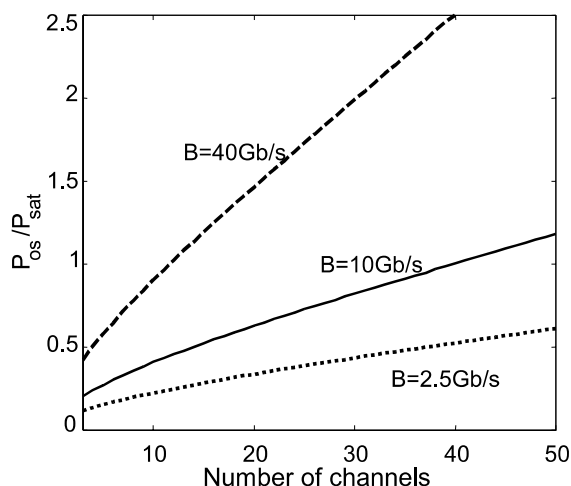


Fig. 4. Allowed output power  $P_a$  for 0.5 dB power penalty versus number of channels. The ordinate is normalized by saturate power  $P_{sat}$ .

The first three figures along with the analytical result (16) obviously show that for a given output power, the power penalty induced by the device will decrease considerably with the increasing bit rate and number of channels. In other words, if the power penalty is confined to a maximum value, the permitted total output power will increase when the bit rate and channel number increase. This means, for some applications, the device can work in a deeply saturated state without significant degradation of the system performance.

Fig. 4 shows the maximum output power allowed for a 0.5 dB power penalty in different conditions. From it, we can see that a high capacity system allows us to use SOA in a deeply saturated state effectively. The former small-signal restriction imposed on the application of SOA is relaxed in such systems.

#### 4.2. Chirp

Similarly, the chirp in different conditions, characterized by the variation of the temporary frequency, can be obtained from Eq. (18). Fig. 5 shows the dependence of the chirp on the number of channels  $N$  and bit rate  $B$ , in the condition that total output power equals the saturate power of

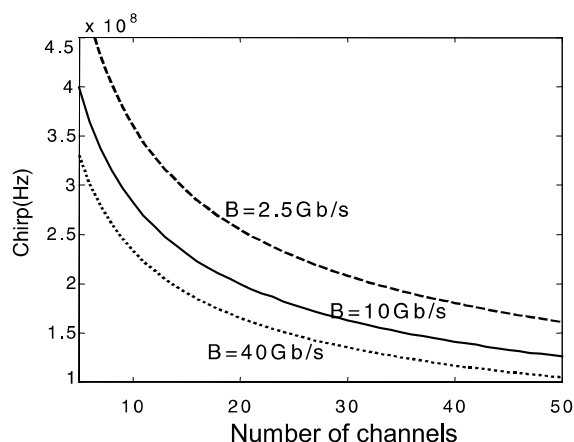


Fig. 5. Chirp induced by SOA versus number of channels in 2.5, 10 and 40 Gbit/s systems.

the device. Obviously, the chirp also decreases along with the increase of the number of channels and bit rate.

### 5. Discussion

In the calculation in Sections 2 and 3, there are mainly three places where approximations are made. These approximations are small-disturbance approximation in Eq. (5), Gaussian distribution approximation of  $\Delta h$  in Eqs. (13) and (14), and Gaussian noise approximation of intensity noise  $\Delta G$  in Eq. (16).

Since the relatively intensity noises calculated in Fig. 1 are in agreement with the numeric simulations, the first two approximations are proved to be valid when the pattern effect is not very severe. Fig. 6 shows the distribution of  $\Delta h$  and  $\Delta G$  of a typical SOA in a  $16 \times 10$  Gbit/s system obtained from the numeric simulations when the output powers are 0.25, and 1 times the saturated power, respectively. Their Gaussian approximation curves are also shown. This figure indicates that the second and third approximations are well obeyed when the pattern effect is not severe and the power penalty is less than 1 dB, so does the whole theory. And in fact, most SOAs used in practical systems will satisfy this condition.

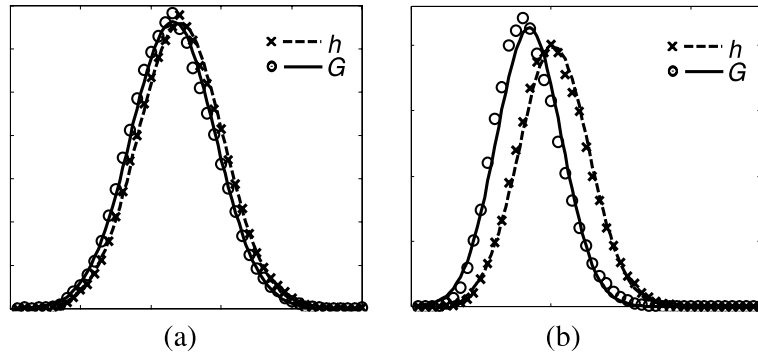


Fig. 6. The marks show the distribution of  $h$  and  $G$  got in the numerical simulations of a  $16 \times 10$  Gbit/s system when the output power of the SOA is (a) 0.25 and (b) 1 times the saturate power. And the curves show the Gaussian distributions with the same average and variance.

For a large output power and severe intensity noise, we can see from Fig. 6 that the distribution of  $G$  decrease faster than Gaussian curves in the lower  $G$  region. Since the influence of intensity noise is more severe in the low gain region, we expect that the power penalty in a real system should be a little less than that obtained in Eq. (16). So, Eq. (16) can be looked on as the upper limit of power penalty in a system with relatively severe pattern effect.

The reported experiments used three or more cascaded SOAs in a transmission system [3–5], and the detailed parameters and working conditions of the SOAs are not fully given, so we can not perform an accurate comparison between the theory and the reported experiments. However, the shapes of the power penalty versus output power curves are identical with the corresponding curves reported in or derived from those experiments.

## 6. Suppression of intensity noise

To suppress intensity noise, Sun et al. proposed and demonstrated the technology of adding a CW reservoir channel with wavelength outside the signal band [3].

Theoretically speaking, this will cause the average value of total power to increase while the variance of it remains unchanged. From Eqs. (6)

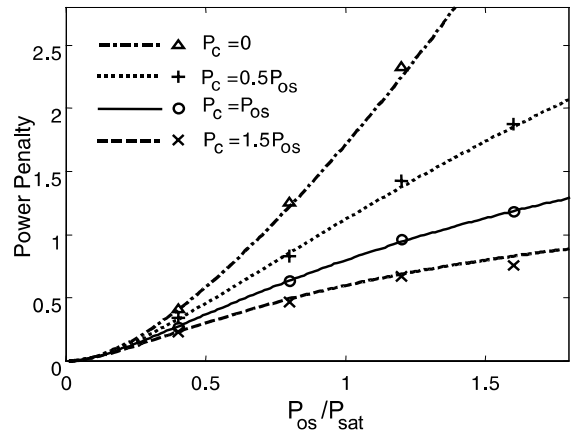


Fig. 7. Power penalty versus total output signal power.  $P_c$  is the output power of the CW channel and  $P_{os}$  is the total output signal power. The numerical simulation results are marked on the figure.

and (17) we can find that the increase of total power will result in a decrease of effective carrier lifetime  $\tau'_c$ , and so, suppress the variance of gain. Consequently, the allowed output signal power will increase for a certain restrained power penalty.

Take a  $32 \times 2.5$  G system as an example. Fig. 7 shows the power penalty versus total output signal power when CW waves of power 0, 0.5, 1 and 1.5 times of the total signal power is injected respectively. The marked points are results from numeric simulations. Numeric studies also show that if the

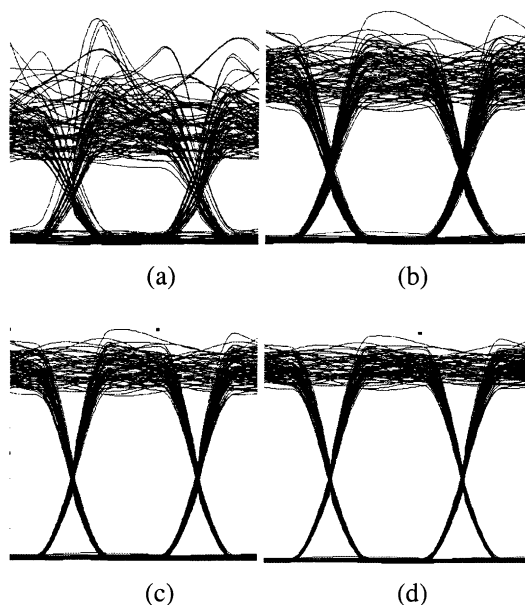


Fig. 8. The simulated eye diagrams for selected conditions with the same output signal power: (a) comes from a  $8 \times 2.5$  GHz system with no CW light; (b), (c) and (d) come from  $32 \times 2.5$  GHz system with CW light power 0, 1, and 1.5 times the signal power respectively.

practical shape of rising and falling edges and finite extinction of the signal is considered, the power penalty are even less for the same average output power. Fig. 8 gives the simulated eye diagrams with the calculated power penalty in several selected conditions.

The noise suppression effect is very obvious when signal power is comparable to the saturation power. However, this effect comes along with the reduction of the gain. So a compromise between the gain and the noise suppression must be made. It seems that a CW wave of power from 1 to 1.5 times the total signal power is recommendable. This corresponds to the experimental setup [3], where the CW power is chosen to be 1.25 times the total signal power.

However, as Eq. (18) shows, the method of injecting CW wave will not suppress the chirp. In fact, the variance of the frequency change will rise a little, due to the faster change of carrier density, as Fig. 9 shows.

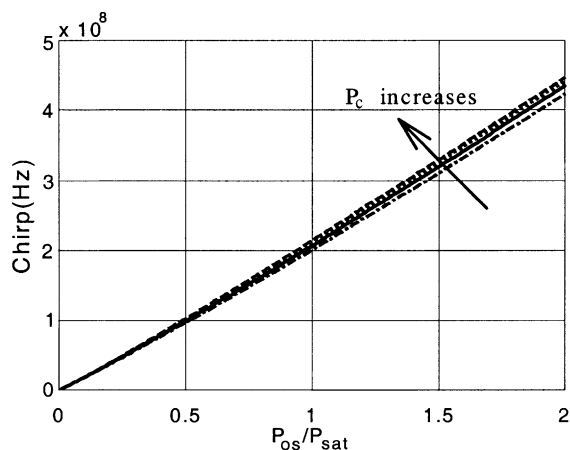


Fig. 9. Chirp induced by SOA versus total output signal power  $P_{os}$  when the CW channel output power  $P_c$  equals 0, 0.5, 1 and 1.5 times the  $P_{os}$ .

## 7. Conclusion

To analyze how SOA can be used as an in-line amplifier in multi-wavelength system, a probability model is established and an analysis result is derived which shows that the noise caused by the pattern effect will decrease with increasing of channel number and bit rate. And this result proves that injection of a CW light is an effective way to further reduce intensity noise. So, for a large capacity system with high speed and many channels, the deterioration of signal quality caused by SOA may be reduced.

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